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TWO-PARAMETER ASYMPTOTIC ANALYSIS OF THE DYNAMICAL EQUATIONS OF THE THEORY OF ELASTICITY FOR THE BENDING OF A PLATE†

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The three-dimensional dynamical equations of the theory of elasticity for the bending of a plate are subjected to asymptotic analysis. Two dimensionless parameters (the exponents of variability and dynamism) characterizing the stress–strain state (SSS) of the plate are varied independently. The asymptotic behaviour of the SSS is established for different parameter values. Cases are found in which the equations of classical plate theory do not furnish a first asymptotic approximation of the equations of the theory of elasticity.

THE EARLIEST attempts to construct a two-dimensional theory of plates [1], by asymptotic integration of the equations of the theory of elasticity, concentrated on the static problem. The characteristic feature of the static formulation is the presence of a small dimensionless parameter—the ratio of the plate thickness to the characteristic length of the strain pattern. In dynamics another small parameter arises—the ratio of the time required by a shear wave to cross the distance between the faces of the plate to the characteristic time scale of the processes being studied. Previous studies devoted to the asymptotic construction of a two-dimensional dynamical theory of plates have usually assumed some relationship between these two parameters [2]—an assumption that *a priori* restricts the range of surface loads that can be considered.

An attempt will be made to develop a general two-parameter analysis of the dynamic bending of plates, on the assumption that the above-mentioned asymptotic parameters are independent. As will be shown, this considerably increases the number of possible asymptotic behaviour patterns for the SSS of the plate.

1. STATEMENT OF THE PROBLEM

The position of a point of the plate in three-dimensional space will be represented by a radius-vector $\mathbf{R} = \mathbf{r}(x^1, x^2) + x^3\mathbf{n}$, where $\mathbf{r}$ is the radius-vector of the middle plane S, $\mathbf{n}$ is the unit vector of the normal to the plane, (x^1, x^2) are curvilinear coordinates on S, and x^3 is the distance from S measured along the normal. Let $a_{\alpha\beta}$ denote the metric tensor of the middle plane S (throughout, Greek indices may take values 1, 2), and let σ^{ij} $(i, j = 1, 2, 3)$ be the stress tensor; the displacement vector $\mathbf{u}$ of the elastic medium will be written as $\mathbf{u} = u^\alpha \mathbf{r}_\alpha + w\mathbf{n}$.

The three-dimensional dynamical equations of the theory of elasticity will then be as follows: the equations of motion

$$\begin{aligned} &\nabla_\alpha \sigma^{\alpha\beta} + \partial\sigma^{3\beta}/\partial x^3 - \rho\partial^2 u^\beta/\partial t^2 = 0 \\ &\nabla_\alpha \sigma^{3\alpha} + \partial\ ^{33}/\partial x^3 - \rho\partial^2 w/\partial t^2 = 0 \end{aligned} \tag{1.1}$$

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the "stress–displacement" formulae

$$
\begin{aligned}
&E\partial w/\partial x^3 = \sigma^{33} - \nu a_{\lambda\mu}\sigma^{\lambda\mu} \\
&E(\nabla_\alpha w + \partial u_\alpha/\partial x^3) = 2(1+\nu)a_{\lambda\alpha}\sigma^{3\lambda} \\
&Ee_{\alpha\beta} = (1+\nu)\sigma_{\alpha\beta} - \nu a_{\alpha\beta}a^{\lambda\mu}\sigma_{\lambda\mu} - \nu a_{\alpha\beta}\sigma^{33} \\
&e_{\alpha\beta} = ½(\nabla_\beta u_\alpha + \nabla_\alpha u_\beta)
\end{aligned}
\tag{1.2}
$$

where ∇_α is the symbol of covariant differentiation, ρ is the density of the plate material, E is Young's modulus, and ν is Poisson's ratio. In addition, we impose the following conditions at the plate faces

$$
\sigma^{33}|_{x^3=\pm h} = \pm Q^3, \quad \sigma^{3\alpha}|_{x^3=\pm h} = Q^\alpha \tag{1.3}
$$

where Q^α and Q^3 are the tensors of the tangential and normal surface loads and $2h$ is the plate thickness.

In (1.3) it is assumed that external forces are applied to both faces in such a way that the only possible SSS of the plate is antisymmetric with respect to the middle plane (bending SSS). More general external loads, other than bending SSSs, may produce SSSs that are symmetric with respect to the middle plane (SSS of extension and transverse compression). In this paper we shall not consider the asymptotic determination of the latter.

We shall assume that the relative half-thickness of the plate $\eta = h/R$ is small (R denotes the characteristic linear dimension in S). Following the scheme of the asymptotic method of [2, 3], we will assume that the independent variables are scaled as follows $[c_s = (E/\rho)^{1/2}]$

$$
x^\alpha = R\eta^q\xi^\alpha, \quad x^3 = R\eta\zeta, \quad t = Rc_s^{-1}\eta^a\tau \tag{1.4}
$$

and that differentiation of any order with respect to the variables ξ^α, ζ, τ does not alter the asymptotic order of the quantities in question.

The numbers q and a in (1.4) are the exponents of variability and dynamism of the SSS, respectively. The exponent of variability characterizes the length of the strain pattern, and the exponent of dynamism characterizes the time rate at which the processes in question take place.

We shall assume that q and a are independent, requiring only that they satisfy the inequalities

$$
q < 1, \quad a < 1 \tag{1.5}
$$

which are necessary conditions for the existence of any two-dimensional plate theory. They imply that we are considering only SSSs in which the characteristic length of the strain pattern significantly exceeds the plate thickness and the characteristic time scale is much longer than the time required by a shear wave to cross the distance between the faces.

The cases most frequently treated in the literature correspond to the following additional restrictions on the exponents q and a: $a \to -\infty$ (statics), and $a = 2q - 1$ (free flexural vibrations). The class of surface loads defined by the condition $a \leq 2q - 1$ was considered in detail in [2].

The asymptotic behaviour of the SSS depends essentially on the form of the surface load. We will first consider the case in which there is no tangential surface load.

2. ASYMPTOTIC INTEGRATION (THE CASE $Q^\alpha = 0$)

We introduce non-dimensional displacements and stresses

$$
\begin{aligned}
&w = Rw^*, \quad u_\alpha = R\eta^{1-q}u_\alpha^* \\
&\sigma^{\alpha\beta} = E\eta^{1-2q-c}\sigma_*^{\alpha\beta}, \quad \sigma^{3\alpha} = E\eta^{2-3q-c}\sigma_*^{3\alpha} \\
&\sigma^{33} = E\eta^{3-4q-b}\sigma_*^{33}
\end{aligned}
\tag{2.1}
$$

where b and c are numbers defined as follows:

$$b = \begin{cases} 0, & a \leqslant 2q-1 \\ 2+2a-4q, & a \geqslant 2q-1 \end{cases}$$
$$c = \begin{cases} 0, & a \leqslant q \\ 2a-2q, & a \geqslant q \end{cases} \tag{2.2}$$

In addition, we shall assume that all quantities marked with an asterisk in (2.1) are of order $O(\eta^{\kappa})$ with the same κ. Then formulae (2.1), (2.2) determine the asymptotic properties of the dynamical SSS of a plate in the bending process.

The equilibrium equations (1.1) and "stress–displacement" formulae (1.2) may be rewritten as follows, taking (1.4) and (2.1) into account

$$\begin{aligned}
&\partial\sigma_*^{3\beta}/\partial\zeta = -\nabla_\alpha^*\sigma_*^{\alpha\beta} + \eta^{2q-2a+c}\partial^2 u_*^\beta/\partial\tau^2 \\
&\partial\sigma_*^{33}/\partial\zeta = -\eta^{b-c}\nabla_\alpha^*\sigma_*^{3\alpha} + \eta^{4q-2a-2+b}\partial^2 w^*/\partial\tau^2 \\
&\partial w^*/\partial\zeta = \eta^{4-4q-b}\sigma_*^{33} - \nu\eta^{2-2q-c}a_{\lambda\mu}\sigma_*^{\lambda\mu} \\
&\partial u_\alpha^*/\partial\zeta = -\nabla_\alpha^* w^* + 2(1+\nu)\eta^{2-2q-c}a_{\lambda\alpha}\sigma_*^{3\lambda} \\
&\sigma_*^{\alpha\beta} = (1-\nu^2)^{-1}\eta^c[(1-\nu)e_*^{\alpha\beta} + \nu a^{\alpha\beta}a_{\lambda\mu}e_*^{\lambda\mu}] + \nu(1-\nu)^{-1}\eta^{2-2q+c-b}a^{\alpha\beta}\sigma_*^{33} \\
&e_{\alpha\beta}^* = ½(\nabla_\beta^* u_\alpha^* + \nabla_\alpha^* u_\beta^*) \quad (\nabla_\alpha^* = R\eta^q\nabla_\alpha)
\end{aligned} \tag{2.3}$$

The operator ∇_α^*, and also the operators $\partial/\partial\xi^1$, $\partial/\partial\xi^2$, do not change the asymptotic orders of the quantities being determined.

Let us look more closely at the system of equations (2.3). The factors η^χ appearing in (2.3) determine the asymptotic order of the individual terms in the three-dimensional equations of the theory of elasticity. Table 1 lists the exponents χ depending on the relationship between q and a. It can be ascertained that, if conditions (1.5) are satisfied, all the powers of η occurring explicitly in (2.3) have non-negative exponents.

First let $a \leqslant 2q-1$. It can be shown that, to within an error $\delta_1 = O(\eta^{2-2q})$, terms in (2.3) corresponding to tangential forces of inertia, the variation of the flexure w with respect to thickness, shear deformation and Poisson's ratio influence of the stress σ^{33} on stresses $\sigma^{\alpha\beta}$ may all be ignored. Thus, to within δ_1, all the assumptions underlying the classical theory of plate bending are valid. It should be noted that the error δ_1 is exactly that predicted by the general Kirchhoff–Love static theory of shells [3].

If $a \geqslant q$, one must, even for the most rough approximation, preserve in (2.3) terms corresponding to tangential forces of inertia and Poisson's influence of σ^{33} on the shear stresses $\sigma^{\alpha\beta}$—the terms normally ignored in classical plate theory. When $a \geqslant q$, the asymptotically subsidiary terms in the system of equations are of order η^{2-2a} (Table 1).

The case $2q-1 < a < q$ is intermediate. Here all the assumptions of the classical theory are satisfied, but the error $O(\eta^{2q-2a})$ that they involve is greater than in statics.

These arguments show that the error of the general dynamical theory of plate bending will depend on two parameters. For the asymptotic theory of the first approximation, this error will be

$$\delta_2 = O(\eta^{2-2q} + \eta^{2-2a}) \tag{2.4}$$

We will now integrate system (2.3) with respect to the transverse coordinate ζ. It can be shown

TABLE 1

χ	$a \leqslant 2q-1$	$2q-1 \leqslant a \leqslant q$	$a \geqslant q$
$2q-2a+c$	$2q-2a(\geqslant 2-2q)$	$2q-2a$	0
$4q-2a-2+b$	$4q-2a-2$	0	0
$4-4q-b$	$4-4q$	$2-2a$	$2-2a$
$2-2q-c$	$2-2q$	$2-2q$	$2-2a$
$2-2q+c-b$	$2-2q$	$2q-2a$	0

immediately that there is a class of three-dimensional SSSs, represented by polynomials in ζ and described to within the error δ_2 by Eqs (2.3) and the face conditions (1.3). This class is defined by formulae

$$\begin{gathered} w^* = w^{(0)}, \quad u_\alpha^* = \zeta u_\alpha^{(1)}, \quad e_{\alpha\beta}^* = \zeta e_{\alpha\beta}^{(1)}, \quad \sigma_*^{\alpha\beta} = \zeta \sigma_{(1)}^{\alpha\beta} \\ \sigma_*^{3\beta} = \sigma_{(0)}^{3\beta} + \zeta^2 \sigma_{(2)}^{3\beta}, \quad \sigma_*^{33} = \zeta \sigma_{(1)}^{33} + \zeta^3 \eta^{b-c} \sigma_{(3)}^{33} \end{gathered} \tag{2.5}$$

in which quantities denoted by an additional index in parentheses are functions of the variables ξ_1, ξ_2, τ or, equivalently, x_1, x_2, t, and are $O(\tau^\kappa)$ with the same κ for all these quantities. They are related through the following equalities

$$\begin{gathered} u_\alpha^{(1)} = -\nabla_\alpha^* w^{(0)}, \quad e_{\alpha\beta}^{(1)} = \tfrac{1}{2}(\nabla_\beta^* u_\alpha^{(1)} + \nabla_\alpha^* u_\beta^{(1)}) \\ \sigma_{(1)}^{\alpha\beta} = (1-\nu^2)^{-1}\eta^c[(1-\nu)e_{(1)}^{\alpha\beta} + \nu a^{\alpha\beta} a_{\lambda\mu} e_{(1)}^{\lambda\mu}] + \\ + \nu(1-\nu)^{-1}\eta^{2-2q+c-b} a^{\alpha\beta}\sigma_{(1)}^{33} \\ \sigma_{(2)}^{3\beta} = -\tfrac{1}{2}\nabla_\alpha^* \sigma_{(1)}^{\alpha\beta} + \tfrac{1}{2}\eta^{2q-2a+c}\partial^2 u_{(1)}^\beta/\partial\tau^2 \\ \sigma_{(0)}^{3\beta} = -\sigma_{(2)}^{3\beta}, \quad \sigma_{(1)}^{33} = -\eta^{b-c}\nabla_\alpha^* \sigma_{(0)}^{3\alpha} + \eta^{4q-2a-2+b}\partial^2 w^{(0)}/\partial\tau^2 \\ \sigma_{(3)}^{33} = -\tfrac{1}{3}\nabla_\alpha^* \sigma_{(2)}^{3\alpha}, \quad \sigma_{(1)}^{33} + \eta^{b-c}\sigma_{(3)}^{33} = Q_*^3 \quad (Q_*^3 = E^{-1}\eta^{4q-3+b}Q^3) \end{gathered} \tag{2.6}$$

The dimensionless load Q_*^3 does not exceed the quantities marked with an asterisk in (2.1) in order of magnitude [this follows from (2.1) and the face conditions (1.3)].

To verify this result, we substitute (2.5) and (2.6) into Eqs (2.3) and into the face conditions (1.3); we then omit quantities of order δ_2 compared with unity throughout the calculations.

3. ASYMPTOTIC INTEGRATION (THE CASE $Q^3 = 0$)

Consider the case in which there is no normal surface load. The asymptotic behaviour of the SSS is described by the equalities

$$\begin{gathered} w = Rw^*, \quad u_\alpha = R\eta^{1-q-c}u_\alpha^* \\ \sigma^{\alpha\beta} = E\eta^{1-2q-c}\sigma_*^{\alpha\beta}, \quad \sigma^{3\alpha} = E\eta^{2-3q-b}\sigma_*^{3\alpha} \\ \sigma^{33} = E\eta^{3-4q-2c}\sigma_*^{33} \end{gathered} \tag{3.1}$$

As before, we assume that the exponents q and a obey inequalities (1.5) and the numbers b and c are defined by (2.2).

Transforming independent variables in the three-dimensional equations of the theory of elasticity (1.1), (1.2) by formula (1.4) and using (3.1), we rewrite the equations as follows:

$$\begin{gathered} \partial\sigma_*^{3\beta}/\partial\zeta = -\eta^{b-c}\nabla_\alpha^*\sigma_*^{\alpha\beta} + \eta^{2q-2a+b-c}\partial^2 u_*^\beta/\partial\tau^2 \\ \eta^{b-2c}\partial\sigma_*^{33}/\partial\zeta = -\nabla_\alpha^*\sigma_*^{3\alpha} + \eta^{4q-2a-2+b}\partial^2 w^*/\partial\tau^2 \\ \partial w^*/\partial\zeta = \eta^{4-4q-2c}\sigma_*^{33} - \nu\eta^{2-2q-c}a_{\lambda\mu}\sigma_*^{\lambda\mu} \\ \partial u_\alpha^*/\partial\zeta = -\eta^c\nabla_\alpha^* w^* + 2(1+\nu)\eta^{2-2q+c-b}a_{\lambda\alpha}\sigma_*^{3\lambda} \\ \sigma_*^{\alpha\beta} = (1-\nu^2)^{-1}[(1-\nu)e_*^{\alpha\beta} + \nu a^{\alpha\beta}a_{\lambda\mu}e_*^{\lambda\mu}] + \nu(1-\nu)^{-1}\eta^{2-2q-c}a^{\alpha\beta}\sigma_*^{33} \\ e_{\alpha\beta}^* = \tfrac{1}{2}(\nabla_\beta^* u_\alpha^* + \nabla_\alpha^* u_\beta^*) \end{gathered} \tag{3.2}$$

It is obvious from (3.2) and Table 2 (which is analogous to Table 1) that the construction of a plate bending theory valid to within δ_2 must make allowance for the transverse shear deformation, but the tangential forces of inertia, the variation of deflection through the thickness of the plate and the Poisson's ratio effect of σ^{33} on $\sigma^{\alpha\beta}$ may be ignored.

TABLE 2

X	$a \leq 2q-1$	$2q-1 \leq a \leq q$	$a > q$
$b-c$	0	$2+2a-4q$	$2-2q$
$2q-2a+b-c$	$2q-2a$ $(>2-2q)$	$2-2q$	$2-2a$
$b-2c$	0	$2+2a-4q$	$2-2a$
$4q-2a-2+b$	$4q-2a-2$	0	0
$4-4q-2c$	$4-4q$	$4-4q$	$4-4a$
$2-2q-c$	$2-2q$	$2-2q$	$2-2a$
$2-2q+c-b$	$2-2q$	$2q-2a$	0

It can be verified that, to within δ_2, system (3.2) has solutions in which the quantities marked with an asterisk in (3.1) satisfy the following equalities

$$
\begin{aligned}
&w^* = w^{(0)}, \quad u_\alpha^* = \zeta u_\alpha^{(1)}, \quad e_{\alpha\beta}^* = \zeta e_{\alpha\beta}^{(1)} \\
&\sigma_*^{\alpha\beta} = \zeta \sigma_{(1)}^{\alpha\beta}, \quad \sigma_*^{3\beta} = \sigma_{(0)}^{3\beta} + \zeta^2 \eta^{b-c} \sigma_{(2)}^{3\beta} \\
&\sigma_*^{33} = \zeta \sigma_{(1)}^{33} + \zeta^3 \sigma_{(3)}^{33}
\end{aligned} \tag{3.3}
$$

which define the variation of the required SSS as a function of the thickness variable ζ. Quantities denoted in (3.3) by added indices in parentheses depend only on ξ^1, ξ^2, τ and are related through the formulae

$$
\begin{aligned}
&u_\alpha^{(1)} = -\eta^c \nabla_\alpha^* w^{(0)} + 2(1+\nu)\eta^{2-2q+c-b} a_{\lambda\alpha} \sigma_{(0)}^{3\lambda} \\
&e_{\alpha\beta}^{(1)} = \tfrac{1}{2}(\nabla_\beta^* u_\alpha^{(1)} + \nabla_\alpha^* u_\beta^{(1)}) \\
&\sigma_{(1)}^{\alpha\beta} = (1-\nu^2)^{-1}[(1-\nu) e_{(1)}^{\alpha\beta} + \nu a^{\alpha\beta} a_{\lambda\mu} e_{(1)}^{\lambda\mu}] \\
&\sigma_{(2)}^{3\beta} = -\tfrac{1}{2} \nabla_\alpha^* \sigma_{(1)}^{\alpha\beta} \\
&\sigma_{(1)}^{33} = \eta^{2c-b}[-\nabla_\alpha^* \sigma_{(0)}^{3\alpha} + \eta^{4q-2a-2+b} \partial^2 w^{(0)}/\partial\tau^2] \\
&\sigma_{(3)}^{33} = -\tfrac{1}{3}\eta^c \nabla_\alpha^* \sigma_{(2)}^{3\alpha} + \tfrac{1}{3}\eta^{2q-2a+c} \partial^2 w^{(2)}/\partial\tau^2 \\
&\sigma_{(1)}^{33} + \sigma_{(3)}^{33} = 0, \quad \sigma_{(0)}^{3\beta} + \eta^{b-c} \sigma_{(2)}^{3\beta} = Q_*^\beta \\
&(Q_*^\beta = E^{-1}\eta^{3q-2+b} Q^\beta, \quad w^{(2)} = -\tfrac{1}{2}\nu a_{\lambda\mu} \sigma_{(1)}^{\lambda\mu})
\end{aligned} \tag{3.4}
$$

4. THE TWO-DIMENSIONAL EQUATIONS IN TERMS OF FORCES AND MOMENTS

We shall derive the two-dimensional equations describing the bending of the plate, in terms of the tensor of moments $M^{\alpha\beta}$, the tensor of transverse shear forces N^β and the flexure of the middle plane W. These quantities are defined as follows:

$$
W = w|_{x^3=0}, \quad M^{\alpha\beta} = \int_{-h}^{h} \sigma^{\alpha\beta} x^3 dx^3, \quad N^\beta = \int_{-h}^{h} \sigma^{3\beta} dx^3 \tag{4.1}
$$

Using the formulae of Secs 2 and 3 and transforming, we obtain equations in terms of W, $M^{\alpha\beta}$, N^α for the cases $Q^\alpha = 0$ and $Q^3 = 0$. Combining these equations so as to retain all terms corresponding to each of these cases, and ignoring quantities of order δ_2, we obtain:

the "moment–flexure" relation

$$
\begin{aligned}
M^{\alpha\beta} = &-\frac{2Eh^3}{3(1-\nu^2)}[(1-\nu)\nabla^\alpha\nabla^\beta W + \nu a^{\alpha\beta}\Delta W] + \\
&+ \frac{2}{3}h^3[\nabla^\alpha Q^\beta + \nabla^\beta Q^\alpha + \frac{2\nu}{1-\nu} a^{\alpha\beta}\nabla_\lambda Q^\lambda] +
\end{aligned}
$$

$$+\frac{2}{3}h^2\frac{\nu}{1-\nu}a^{\alpha\beta}Q^3 \quad (\Delta = a^{\lambda\mu}\nabla_\lambda\nabla_\mu) \tag{4.2}$$

the equations of motion

$$\begin{aligned} &\Delta_\alpha M^{\alpha\beta} - N^\beta + 2hQ^\beta + {}^2\!/_3\rho h^3\nabla^\beta\partial^2 W/\partial t^2 = 0 \\ &\nabla_\alpha N^\alpha + 2Q^3 - 2\rho h\partial^2 W/\partial t^2 = 0 \end{aligned} \tag{4.3}$$

These equations differ from those of classical plate theory in that formulae (4.2) include terms that depend on the surface loads Q^α and Q^3; in addition, the moment equation of motion (4.3) contains an inertial term. To within δ_2, we can apply the following change of variables in the equation:

$$ {}^2\!/_3\rho h^3\nabla^\beta\partial^2 W/\partial t^2 \to {}^2\!/_3 h^2\nabla^\beta Q^3$$

Situations in which these "non-classical" terms become asymptotically significant may be described using the arguments of Secs 2 and 3.

Expressing the transverse shear forces and moments in the equations of motion in terms of W and dropping terms of order δ_2, we obtain the resolvent equation of classical plate theory:

$$ {}^2\!/_3(1-\nu^2)^{-1}Eh^3\Delta^2 W + 2\rho h\partial^2 W/\partial t^2 = 2Q^3 + 2h\nabla_\lambda Q^\lambda \tag{4.4}$$

Note that if the condition $|Q^3 + h\nabla_\lambda Q^\lambda| \ll |Q^3|$ is satisfied, the relative asymptotic order of the flexure may be less than dictated by the asymptotic formulae (2.1) and (3.1). For example, it may happen that $w \sim u_\alpha$. SSSs possessing the above asymptotic forms are characteristic for plates with non-classical boundary conditions on the faces [4, 5].

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